Physica A 392 (2013) 3731–3740

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Complex dynamics in the distribution of players' scoring performance in Rugby Union world cups

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HIGHLIGHTS

- Players' scoring performance was investigated over 7 rugby world cups (1987–2011).
- A special attention was given to the switch between amateurism and professionalism.
- The simplified canonical law describes well the distribution of points scored.
- Professionalism led to a decrease in the relative performance of best scorers.
- Professionalism led to a progressive homogenization among all scorers performance.

ARTICLE INFO

Article history:
Received 25 September 2012
Received in revised form 11 January 2013
Available online 17 April 2013

Keywords:
Sport
Performance indicator
Simplified canonical law
Rank-frequency
Diversity
Evenness

ABSTRACT

The evolution of the scoring performance of Rugby Union players is investigated over the seven rugby world cups (RWC) that took place from 1987 to 2011, and a specific attention is given to how they may have been impacted by the switch from amateurism to professionalism that occurred in 1995. The distribution of the points scored by individual players, P_s, ranked in order of performance were well described by the simplified canonical law $P_s \propto (r+\phi)^{-\alpha}$, where r is the rank, and ϕ and α are the parameters of the distribution. The parameter α did not significantly change from 1987 to 2007 ($\alpha = 0.92 \pm 0.03$), indicating a negligible effect of professionalism on players' scoring performance. In contrast, the parameter ϕ significantly increased from $\phi = 1.32$ for 1987 RWC, $\phi = 2.30$ for 1999 to 2003 RWC and $\phi = 5.60$ for 2007 RWC, suggesting a progressive decrease in the relative performance of the best players. Finally, the sharp decreases observed in both $\alpha(\alpha = 0.38)$ and $\phi(\phi = 0.70)$ in the 2011 RWC indicate a more even distribution of the performance of individuals among scorers, compared to the more heterogeneous distributions observed from 1987 to 2007, and suggest a sharp increase in the level of competition leading to an increase in the average quality of players and a decrease in the relative skills of the top players. Note that neither α nor ϕ significantly correlate with traditional performance indicators such as the number of points scored by the best players, the number of games played by the best players, the number of points scored by the team of the best players or the total number of points scored over each RWC. This indicates that the dynamics of the scoring

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http://dx.doi.org/10.1016/j.physa.2013.03.024

performance of Rugby Union players is influenced by hidden processes hitherto inaccessible through standard performance metrics; this suggests that players' scoring performance is connected to ubiquitous phenomena such as anomalous diffusion.

1. Introduction: the game of rugby

Rugby Union, one of the two codes of rugby football, was declared as an open game, free of the previous restriction of amateurism, on 26 August 1995 by the International Rugby Football Board, which later became the International Rugby Board (IRB). The Rugby World Cup, first held in 1987 in New Zealand, takes place every four years, and is one of the major international rugby competitions with the Six Nation Championship in Europe (disputed by England, France, Ireland, Italy, Scotland and Wales), and the Rugby Championship in the Southern Hemisphere (disputed by Argentina, Australia, New Zealand and South Africa). Note that the Six Nations Championship and the Rugby Championship are the successors of the Five Nations Championship and the Tri Nations, which have respectively been contested between England, France, Ireland, Scotland and Wales from 1940 to 1999, and between Australia, New Zealand and South Africa from 1996 to 2011. Readers unfamiliar with the game may refer to http://en.wikipedia.org/wiki/Rugby_union and http://www.irb.com for details on the origin, rules and governing bodies.

To date, several aspects of the game have been thoroughly investigated, including game statistics [1–4], players skills [5,6], player characteristic and game patterns [7–9], the effects of rule changes on game patterns [10–13] and on the incidence of injury [14], and different aspects of performance analysis [15] such as the physical demands of performance [16–19], the tactical aspects of performance [20–23], and the technical analysis of performance [24].

However, despite a few reviews on the social and financial consequences of Rugby Union professionalism in Australia, New Zealand and South Africa [25–29], considerations of the consequences of the shift between amateurism and professionalism in Rugby Union have been limited to studies of the incidence of injuries [29–31] and playing patterns [3,32]. Due to an expanding consumer culture (and despite early warnings that if the code was to pay its players and administrators it would transform from play into work and destroy its amateur ethos and character building quality [33]), the traditional cultural and social barriers that were thought to divide business from sport vanished [34]. Commercialization overwhelmed the game in 1995 when Murdoch's global TV network entered into the multimillion agreement with the national governing bodies of Australia, New Zealand and South Africa. As a consequence, Rugby Union is not only a sport anymore, but became a business that craves for media attention, corporate support and audience interest, and subsequently increased the demand for televised rugby competitions and the derived demand for rugby player [25]. Rugby Union transformed into a highly professionalized and business-like activity, with players earning massive salaries and receiving a lot of media attention such as New Zealander Jonah Lomu and Englishman Jonny Wilkinson, who were the first Rugby Union players to reach a status of global superstars.

No attention has been given, however, to the individual scoring performance of players over the years, and how they might have been impacted by professionalism. In this context, this paper shows that the distribution of rugby players individual scoring performance in seven successive world cups is well adjusted by the Zipf–Mandelbrot scaling law [35,36]. It also investigates how players relative performance may have been impacted by professionalism, and discusses their origin and significance through an analogy with the economic and ecological systems, the system of ideal gases, and idealized theoretical systems where individuals are rewarded not according to their absolute performance, but according to their performance relative to others, and coined 'celebrity markets' [37].

2. Complex systems and the simplified canonical law

Many natural complex systems such as rainfalls [38,39], earthquakes [40,41], solar flares [42], population growth [43], species lifetimes [44] and abundance [45], population dynamics [46], forest fires [47], rainforest dynamics [48] and epidemics [49]. This has also been observed in social and economic systems such as city size [50–52] and growth [53,54], company size [55] and income [56], the sales of music recordings [57], scientific citations [58,59], internet surfing [60,61], the frequency of occurrence of personal names in most cultures [62], car traffic [63] and war distribution [64,65] that all are characterized by rare large fluctuations interspersed among long period of relative stasis. For instance, at the evolutionary scale, adaptive radiations occur in short bursts of evolutionary activity, referred to as punctuated equilibrium [66,67], and extinction events in the fossil records appear episodic with relatively long periods of stability alternating with short-lived extinction events [68]. Biomass and species are also rarely dispersed uniformly, but instead characterized by intermittent distributions, with rare dense patches interspersed among a wide range of low density patches [69]. Similarly, a long text generally contains many words that are employed in few contexts and a small number that occurs widely [70,71]. These systems typically self-organize to produce scale-free distributions [72,73]. More recently heavy-tailed distributions were also found in the statistical distribution of achievement in sport and science [74,75].

Scale-free distributions were recognized more than a century ago by the economist Vilfredo Pareto (1848–1923) who showed that the wealth w in any society followed the power-law form $P(W) \propto w^{-\gamma}$ [76], or equivalently $P(W > w) \propto$

$w^{-\beta}$, where W is a wealth threshold value and $\gamma = \beta + 1$ [69]. Similarly, the linguist George K. Zipf (1902–1950) observed that the frequency of occurrences of words in a variety of languages were described by the so-called Zipf's law which states that the frequency f_r of the rth most frequent word is inversely proportional to its rank r as $f_r = f_1/r$, where f_1 is the frequency of the most frequent word in the distribution [70,71]. While the Zipf's law describes remarkably well the distribution of words in a variety of languages [70,71], the generalized Zipf's law

$$f_r \propto r^{-\alpha} \tag{1}$$

was introduced to describe the divergence from the original Zipf's law. If $\alpha > 1$, rare words are used relatively more often; this is actually seldom found, examples being modern Hebrew and the English of the Pennsylvania Dutch [36]. In contrast, $\alpha < 1$ corresponds to the largest variety of verbalization. As α decreases, common words are used more and more frequently, with very low values (typically $\alpha \leq 0.6$) being associated with schizophrenics [36]. Note that, the exponent α (Eq. (1)) is connected to the Paretian power-law exponents β and γ as $\alpha = 1/\beta$ and $\alpha = 1/(\gamma - 1)$ [69].

Based on the condition that, for a given quantity of information to be conveyed per word, the cost per word is to be minimized, *i.e.* the Principle of Least Effort initially introduced by G.K. Zipf [71], Eq. (1) appears as a special case of a more fundamental relation, the so-called simplified canonical law (SCL hereafter) defined as [76]:

$$f_r \propto f(r + \phi)^{-\alpha} \tag{2}$$

where $f, f^{-1} = \sum_{r=1}^{N} (r + \phi)^{-\alpha}$, is a normalizing coefficient with N being the number of words, r their rank, and ϕ a fitting parameter describing the deviation of the function f_r from a pure power law (*i.e.* where $\phi = 0$) over the highest rank. Note that Eq. (2) revealed that instead of all languages being characterized by $\alpha = 1$ and $\phi = 0$, as implied by Eq. (1), there were considerable differences between them [36].

Considering that words are the medium with which thoughts are given expression, players may be considered as the medium with which the desire of achievement of their teams, such as scoring and winning, are satisfied. In this context, this work aims (i) to investigate whether the distributions of players' scoring performance during Rugby Union world cups can be described by the simplified canonical law, (ii) to assess whether different world cups are characterized by different values of the parameters α and ϕ, with a specific focus on the switch from amateurism to professionalism, and (iii) to establish the significance of these parameters in the competitive world of sports using analogies with economic and ecological systems.

3. Rugby players' scoring performance as a complex system

Scale-free distributions have been observed in sports, for instance in soccer championships [77] and men's world track records for running distances ranging from 100 m to 200 km [78], where there are respectively many players that make few goals in contrast to the best scorers and very few fast runners among a range of slow runners. It is hence likely that among world cup rugby players, many players score few points in contrast to top scorers. To investigate this question, we used the statistics of the best 100 scorers of the seven rugby world cups that took place every four years since 1987 (available through the International Rugby Board website, www.rugbyworldcup.com). The disparity of scoring performances between world cups (Fig. 1) indicates that the role played by the best scorers is independent of their absolute performance and the performance of their teams in terms of overall scoring performance and world cup ranking. This is a simple analogy with financial markets, where individuals are paid not according to their absolute performance, but according to their performance relative to others in the same business. Hence, the points scored by each of the best 100 scorers P_s were ranked and used to assess how well they fit the simplified canonical law

$$P_s \propto (r + \phi)^{-\alpha} \tag{3}$$

for each world cup (Fig. 2). The parameters α and ϕ were estimated for each world cup using a nonlinear least-squares Levenberg–Marquardt algorithm, and were chosen as the values that respectively maximized and minimized the coefficient of determination r^2 and the sum of the squared residuals between empirical data and Eq. (3). Note that Eq. (3) provides a very good fit to the entire empirical distribution of players' scoring performance for the RWC held from 1987 to 2007. In contrast, it only fits the performances of the best first 46 scorers in RWC 2011, which is uniquely characterized by a very sharp decay of P_s for the lowest ranks and a high proportion of ex-æquo (Fig. 2). The fitted parameters are shown in Fig. 3 and Table 1, and led to discriminate Rugby Union world cups (RWC) into 4 distinct groups. The first three groups respectively include the world cups held in 1987 (group 1, G_1), in 1991 and 1995 (group 2, G_2) and in 1999, 2003 and 2007 (group 3, G_3). These groups are characterized by non-significantly different values ($p > 0.05$) of the parameter α ($\alpha = 0.92 \pm 0.03$; $\bar{x} \pm$ SD). In contrast, the values of ϕ increase from $\phi = 1.32$ for G_1, $\phi = 2.30$ for G_2, and $\phi = 5.60$ for G_3. The fourth group (G_4) corresponds to the 2011 RWC, which is very distinctive with values of α ($\alpha = 0.38$) and ϕ ($\phi = 0.70$) that are both highly significantly smaller than the values estimated for the six previous world cups ($p < 0.01$). Note that neither α nor ϕ significantly correlate ($p > 0.05$) with traditional performance indicators such as the number of points scored by the best players, the number of games played by the best players, the number of points scored by the team of the best players or the total number of points scored over each RWC. This indicates that the dynamics of the scoring performance of Rugby Union players is influenced by hidden processes hitherto inaccessible through standard performance metrics; this suggests that players' scoring performance is connected to ubiquitous phenomena such as anomalous diffusion.

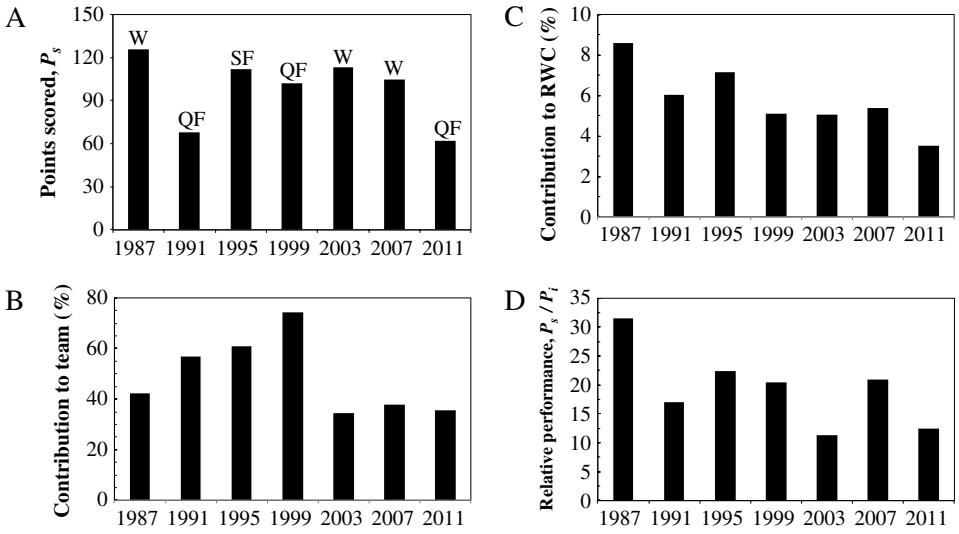

Fig. 1. Absolute and relative scoring performances of the best scorers of each of the seven rugby world cups, *i.e.* G. Fox (New Zealand; 1987), R. Keyes (Ireland; 1991), T. Lacroix (France; 1995), G. Quesada (Argentina; 1999), J. Wilkinson (England; 2003), P. Montgomery (South Africa; 2007) and M. Steyn (South Africa; 2011), that were respectively won by New Zealand, Australia, South Africa, Australia, England, South Africa and New Zealand. Absolute scoring performances were expressed as (A) the total number of points P_s scored (the letters at the top of bar graphs indicate the level of the competition reached by the teams of the best scorer). The relative scoring performances are expressed as the contribution (%) of the best scorers to the total number of points scored by their respective teams (B) and to the total number of points scored during each RWC (C), and as the ratio P_s/P_i, where P_i is the smallest number of points scored by a player in their respective teams, and (H). W: winner; SF: semi-finalist; QF; quarter-finalist.

Table 1
Best fitting parameters of the simplified canonical law (SCL), $P_s \propto (r+\phi)^{-\alpha}$, where P_s is the number of points scored by each of the top 100 scorers of each rugby world cup (RWC) from 1987 to 2011, and the parameters α and ϕ are estimated using a nonlinear least-squares Levenberg–Marquardt algorithm, and are chosen as the values that respectively maximized and minimized the coefficient of determination r^2 and the sum of the squared residuals (SSR) between empirical data and the simplified canonical law. Note that the SCL only fits the performances of the best first 46 scorers in RWC 2011, which is uniquely characterized by a very sharp decay of P_s for the lowest ranks and a high proportion of ex-æquo.

RWC			r^2	SSR
1987	0.91	1.32	0.95	0.400
1991	0.92	2.30	0.97	0.361
1995	0.97	2.30	0.98	0.292
1999	0.88	5.50	0.96	0.436
2003	0.93	5.50	0.98	0.154
2007	0.93	5.80	0.95	0.527
2011	0.38	0.70	0.96	0.015

To investigate further the dynamics observed among RWC, the distance between two successive RWC was estimated as [79]:

$$r_{\mathrm{RWC}_{i,j}} = \left[\frac{1}{N}\sum_{k=1}^{N}\left(r_{\mathrm{RWC}_i}(P_k) - r_{\mathrm{RWC}_j}(P_k)\right)^2\right]^{1/2} \tag{4}$$

where $r_{\mathrm{RWC}_i}(P_k)$ and $r_{\mathrm{RWC}_j}(P_k)$ are the ranks of all players that scored in both rugby world cups i and j. Both the distance $r_{\mathrm{RWC}_{i,j}}$ (Fig. 4(a)) and the number of players who scored during two consecutive RWC (Fig. 4(b)) did not exhibit any pattern that could be related to the switch between amateurism and professionalism, or with any of the various performance indicators discussed above (see Fig. 1). This is another indication of the highly dynamic nature of the processes behind players' scoring performance, and it suggests that the scoring dynamics of Rugby Union world cups may be controlled by the potentially conflicting, hence complex, interactions between:

(i) the role played by the highly variable level of performance of most players consistently scoring during successive world cups; for instance, among players who played 2 or more rugby world cups, very few were consistently scoring such as Michael Lynagh from Australia (who ranked 2nd in 1987 and 1991 and 6th in 1995), and Gavin Hastings from Scotland (who ranked 3rd in 1987 and 1991 and 2nd in 1995). Most other players exhibited highly variable levels of performance such as *e.g.* the Frenchman Thierry Lacroix who ranked 24 and 1 in 1991 and 1995 RWC, the South African Josh Kronfeld who ranked 22nd in 1995 but only 98th in 1999, and the English winger Rory Underwood who ranked 49, 20 and 14 during the 1987, 1991 and 1995 RWC. Even Jonny Wilkinson, the most prolific scorer in the history of the Rugby World Cup, went

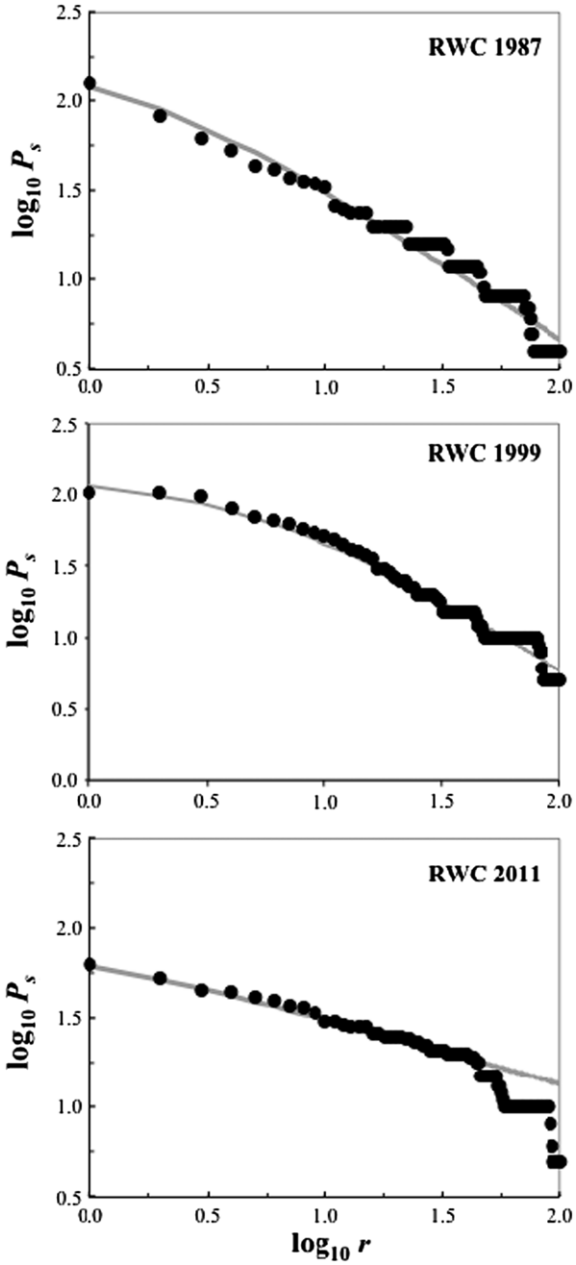

Fig. 2. Double logarithmic plots of the point scored by individual players P_s versus player rank r during the 1987, 1999 and 2011 Rugby World Cups. The black dots are the empirical data, and the continuous gray line their best fit by the Mandelbrot simplified canonical law $P_s \propto (r+\phi)^{-\alpha}$.

down from a top scorer status in his first three RWC (rank 5 in 1999, 1 in 2003, and 3 in 2007 RWC) to a relatively mediocre 15th rank in 2011. Note that, the number of points scored by the top players did not correlate with the number of games they played. For instance, the number of RWC games played by Jonny Wilkinson over his 12 years RWC career (4, 6, 4 and 4 games in 1999, 2003, 2007 and 2011) did not impact his scoring ability (69, 113, 67 and 28 points in 1999, 2003, 2007 and 2011), hence suggests a minimal impact of a player's age on his level of performance. If the success of a player depends on many factors such as an individual's talent, productivity and reputation [74], and the interplay between technology and ecophysiological limits [80–82], the individual variability observed in the performance of high longevity players like Jonny Wilkinson might also be related to the increase in the overall level of competition in the rugby game.

(ii) the very irregular number of players appearing in the best ten scorers over the two consecutive world cups; from that point of view, note that the 2003 and 2011 RWC 2011 are distinctive from the previous ones as respectively 9 and 8 of the 10 best scorers did not appear in the top 10 of the previous RWC in contrast to 1991 (5), 1995 (5), 1999 (4) and 2007 (6).

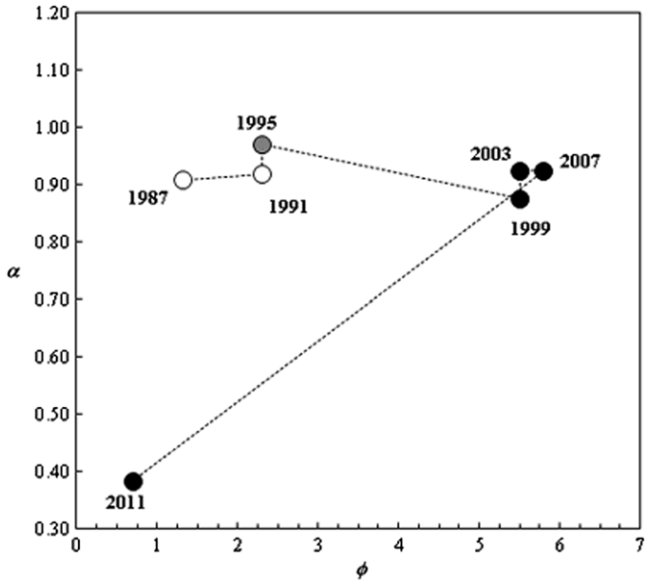

Fig. 3. The fitting parameter α and ϕ of the simplified canonical law, $P_s \propto (r+\phi)^{-\alpha}$ where P_s is the number of points scored by a player and r its rank, estimated for each of the seven rugby world cup played every four years since 1987. The open, gray and black dots respectively correspond to the amateur era, the year rugby turned into a professional game (note, however, that Rugby Union became a professional game on 26 August 1995, two months after the final of the third world cup held in Johannesburg on June 24th, 1995) and the professional era.

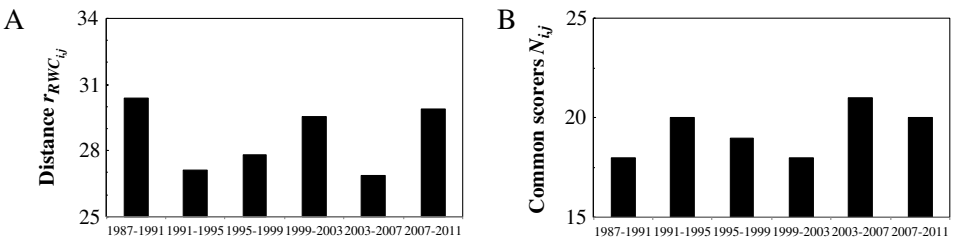

Fig. 4. Distance $r_{\mathrm{RWC}_{i,j}}$ (A) between the ranks of the $N_{i,j}$ players (B) who scored during two successive rugby world cups i and j. The distance between two successive RWC was estimated as $r_{\mathrm{RWC}_{i,j}} = \left[\frac{1}{N}\sum_{k=1}^{N}\left(r_{\mathrm{RWC}_i}(P_k) - r_{\mathrm{RWC}_j}(P_k)\right)^2\right]^{1/2}$ [79], where P_k is the rank of all players that scored in both rugby world cups i and j.

(iii) the localized influence of high performing newcomers such as Percy Montgommery from South Africa and Morney Stein from South Africa who respectively ranked first in the RWC 2007 and 2011, but did not participate to the previous RWC.

(iv) the considerable number of players who did score only during one world cup. Note that this feature, together with the fact that many players only play one RWC, is likely related to the extremely right-skewed longevity distribution for professional athletes [74], especially in a high-contact sport such as rugby.

The influence of the two previous categories of players on the overall RWC scoring dynamics is supported by the significant ($p < 0.05$) negative correlation between the distance $r_{\mathrm{RWC}_{i,j}}$ (Fig. 4(a)) and the number of players who scored during two consecutive RWC (Fig. 4(b)).

4. Rugby players' scoring performance: phenomenological analogies

The simplified canonical law is expected to emerge from a minimizing principle, the so-called Principle of Least Effort [71], which suggests that human speech and language are structured optimally as a result of two opposing forces: uniformity and diversity. If a repertoire is too unified or repetitive, a message is represented by only a few signals and, therefore, less communication complexity is conveyed. Too much uniformity hence makes a sentence unintelligible; if a repertoire is too diverse, less communication is conveyed. These two opposite forces result in a balance between unification and diversification. A vocabulary should not be too poor or too rich, it must be in some kind of equilibrium state, representing a balance between the repetition desired by the listener, and the diversity desired by the transmitter. Just as words are the medium with which thoughts are given expression, in economy and ecology, companies and species are respectively the media with which human economic desires and animal basic needs are satisfied. The SCL distributions observed in company

size [55] and species abundance in ecological systems [83] hence indicate that economic and ecological systems are similarly organized with regards to the allocation of resources. By analogy, players may be thought as the medium with which the performance and success of a rugby team are achieved. The emergence of SCL from the distribution of rugby players' scoring performances may hence be thought as the expression of the fact that the game of Rugby Union is generally organized (at least in the world cup framework) with qualitative efficiency regarding the allocation of resources. However, Fig. 3 shows that there are quantitative differences in the way this efficiency is realized from one world cup to another.

More specifically, the players' scoring performance P_s in the pre-professionalism era (1987, 1991, and *sensu stricto* 1995) and most of the professionalism era (1999, 2003, 2007) is characterized by non-significantly different values of the parameter α, indicating that the overall level of performance of players relative to each other was not impacted by the shift from amateurism to professionalism. Noticeably, the 3-fold decrease observed in the parameter α from the period 1987–2007 to 2011 indicates a homogenization of the player scoring performance relative to each other. In other words, while the relative differences between the scoring performances of players did not significantly evolved from 1987 to 2007, it considerably decreased in 2011. This indicates a more even distribution of the performance of individuals among scorers in 2011, compared to the more heterogeneous distributions observed from 1987 to 2007. In this context, Fig. 3 indicates that the relative skills of the top players have drastically gone down in 2011, suggesting an increase in the average quality of players, hence an increase in the level of competition expected from player dilution which occurs when a league expands. This increase in the competition level is consistent with the individual variability in the performance of high longevity players such as Jonny Wilkinson who scored an average of 83 points in 1999, 2003 and 2007 RWC (*i.e.* 69, 113 and 67, respectively) under similar conditions of competition, but only 28 in 2011 when the level of competition drastically increased.

A simple analogy can be made with ecosystems. Eq. (2) has been used to describe the dynamics of species in both aquatic and terrestrial ecosystems, where f_r is the frequency of the rth species after ranking the species in decreasing order of their frequency [83–87]. Thus, in Eq. (2) the parameters α and ϕ characterize the species diversity and the evenness of a given community; the diversity is given by

$$H = -\sum_{i=1}^{N} f_i \log_2 f_i \tag{5}$$

and the evenness by

$$R = H/\log_2 N \tag{6}$$

where f_i is the relative frequency of the species i and N the number of species. Eq. (6) shows that for the same number of species, the diversity is high when species have equivalent probability (high evenness), and low when few species are frequent and others are scarce (low evenness). The high values of α observed from 1987 to 2007 and the low value observed in 2011, indicate that scorer performances were more evenly distributed in 2011 RWC. This may be related and explained by the dynamics observed in the so-called celebrity markets (*i.e.* occupations like medicine, journalism, share trading and entertainment industries such as sports and film industries where individuals are rewarded not according to their absolute performance, but according to their performance relative to others), hence where '*winners take all the market*' [37]. Specifically, in an attempt to model celebrity markets, the wealth w of N agents investing an equal amount in the market and paid as a function of the good and bad choices they make in the market has been shown to follow a Paretian distribution $P(W) \propto w^{-\gamma}$ [37]. Note that when $\gamma > 2$ ($\alpha > 1$), $\langle w \rangle$ diverges and condensation occurs, *i.e.* when a few agents (ultimately one) are so smart that they dominate the market, hence the concept of super celebrity [37]; see also Refs. [88–90] for more details on condensation in economic systems. For instance, the results of the global stock game played by a group of students as a learning exercise and a group of brokers to get experienced in the real stock market without losing money respectively led to distinct value of the exponent γ, *i.e.* $\gamma = 2.10$ and $\gamma = 1.43$ [37]. The first group consisted of beginners, hence some learners are smarter than others and play well enough to become a super celebrity, $\gamma > 2$. However, the second group consists of experts; hence nobody gains exceptionally different from others, which explains the smaller value of γ. In international Rugby Union, the value of the parameter α are consistently smaller than 1 (Fig. 3), *i.e.* $\gamma < 2$. This shows that none of the best RWC scorers were ever prominently better than others, in agreement with observations conducted on other professional sports such as international cricket where $\gamma < 2$ [37], hence there is no exceptional Rugby Union player who could score strikingly different from others. Note, however, that this statement should be considered with caution considering the considerable difference in the number of games played in international crickets and international Rugby Union. By analogy with linguistics, the scoring performance of Rugby Union players became abnormally evenly distributed during the 2011 RWC, with a pathologically low value of α, *i.e.* schizophrenically low *sensu* Mandelbrot (1954) with $\alpha < 0.6$ [36], indicative of a relative unification of players performances.

The parameter ϕ describes a deviation from a pure Zipf's law for the best scorers. The increase observed in the parameter ϕ from pre-professionalism towards the professionalism era, *i.e.* $\phi_{G_1} < \phi_{G_2} < \phi_{G_3}$ (Fig. 3), indicates that (i) the performance of the best amateur scorers relative to other amateur players were higher during the first RWC, than during the end of the amateurism era in 1991 and 1995 ($\phi_{G_1} = 2.30$) and (ii) that the advent of professionalism led to an additional decrease in the relative difference observed between the performance of the best professional scorers ($\phi_{G_3} = 5.60$). The convex shape of Eq. (3), *i.e.* $\phi > 0$ (Fig. 2), and the increase in the value of the parameter ϕ (Fig. 3), have also been observed in the distribution of the abundance of species in terrestrial and aquatic ecosystems [83–87] and in the distribution of the size of

companies in a range of countries [55]. In ecology, this indicates a more even distribution among most abundant species (*i.e.* a more mature community), resulting in a greater value of evenness among the most frequent species and a higher diversity index [83,86,87]; the most frequent species hence exploit different ecological niches *sensu* Hutchinson [91]. Similarly, in economy, this shows that there are companies of similar size occupying different niches, implying that these niches are in different areas of activity. In the Rugby Union world cup framework, this suggests the competitiveness of the best scorers relative to other players consistently decreased from 1987 to 2007, leading to an increased evenness in the distribution of the scoring performance of the best players. Finally, the 8-fold decrease observed between the first world cups of the professional era (1999, 2003 and 2007; $\phi_{G_3} = 5.60$), and the most recent one (2011; $\phi_{G_4} = 0.70$) indicates a drastic change in the relative performance of the best players. Specifically, as ϕ decreases (ultimately $\phi = 0$), a system contains enough niches for all species, companies or players; hence competition is tolerated. Hence, in the RWC framework, this suggests that the evenness of the scoring performance of the best players sharply decreased from 2007 to 2011. Note that $\phi = 0$ is typically observed at the end of an ecological succession when the first ranked species become more dominant and the species richness is lower, and characterizes a senescent community. This may also suggest, however, that the game of rugby has progressively evolved towards a different dynamics especially during the 2011 RWC.

A negative value of ϕ typically describes a pioneer community marked by the dominance of a few (even one) species that dominate due to their fast growth and reproduction rates in a low species richness and a low evenness assemblage [83]. This has been reported as the '*king effect*' in the distribution of populations in cities [92–94]. A '*king effect*' typically occurs because a few (ultimately one) cities in some countries, by a specific cause (*e.g.* economic, political), play an irregular competition to attract people and do not follow the same rule that most of the cities do. This shows that in both the amateurism and professionalism era, no player has ever overly dominated the competition based on his scoring performance. This may be a limitation related to the intrinsic high competitiveness of the international rugby which *naturally selects* players that are already highly successful at the club and/or province levels, hence the difference between the scoring performances of two international players already having high scoring performance in their respective national competitions is likely to be small. Note that the super-dominant '*king*' or '*super celebrity*' effect was recently shown to emerge in a competition model if the rich-get-richer effect was too large or if the appraisal timescale was too short in both science and professional sports [95].

5. Conclusion

The variability observed in the parameters α and ϕ of the simplified canonical law $P_s \propto (r + \phi)^{-\alpha}$ that best fit the empirical distribution of the points, P_s, scored by individual players during the seven successive rugby world cups held between 1987 and 2011 shows (i) that the switch from amateurism to professionalism has mainly been characterized by a decrease in the relative performance of the best scorers during each RWC (*i.e.* an increase in ϕ), leading to a more even performance distribution among the best scorers, and (ii) a progressive evolution of the game towards a more spread distribution among all RWC scorers, which culminates in 2011 RWC. Both observations are consistent with an increase in the intrinsic high competitiveness and selectivity of the game of Rugby Union at the international level, which is suggested to act as *natural selection* from the national towards the international levels. Specifically, the fact that during both the amateurism and professionalism eras, no player has ever dominated the competition based on his scoring performance suggests that international rugby *naturally selects* players that are already highly successful at the club and/or province levels, hence the difference between the scoring performances of two international players already having high scoring performance in their respective national competitions is likely to be small, and this difference progressively decreased from 1987 to 2011. Note that the switch from amateurism to professionalism did not lead to drastic changes in player performance in Rugby Union world cups, in comparison to those observed between 2007 and 2011. Institutional changes have been observed to have an effect on the player performance in other sports such as baseball via rule change or the use of performance enhancing drugs by a large number of players [96]. Hence, other factors that may be related to the interplay between technology and ecophysiological limits [80–82], performance management [16–24], and eventually the patterns of drug and alcohol consumption [97,98] and the evolution of both genotypes and phenotypes in professional rugby players [99] might need to be explored further to drivers of player performance in RWC.

More generally, the results observed in the present work can be linked to statistical mechanics through two main analogies. First, the deviation from a Zipf's law related to the parameter ϕ is analogous to the deviation of the Fermi–Dirac distribution from the Maxwell–Boltzmann distribution on the assumption that two particles cannot occupy the same state [35]. As such, it is stressed that the basic components of an ecosystem (species), an economy (companies) or a given sport (player) can all be considered as fermions in a statistical mechanics sense. Second, because neither α nor ϕ significantly correlate with the traditional performance indicators, the dynamics of the scoring performance of Rugby Union players is likely to be influenced by the hidden factors hitherto inaccessible through standard performance metrics; this suggests that players' scoring performance is connected to ubiquitous phenomena such as anomalous diffusion.

Acknowledgments

The International Rugby Board (IRB) is acknowledged for making the statistics of the Rugby World Cups freely available through its website (http://www.irb.com). This research was supported under Australian Research Council's Discovery

Projects funding scheme (project number DP0988554). Professor Seuront is the recipient of an Australian Professorial Fellowship (project number DP0988554). Two anonymous referees are acknowledged for their constructive comments and criticisms, and Dr. Alexander Petersen for stimulating discussions on the subject.

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